BIRKHÄUSER-GRÄTZER PRIZE

The *Birkhäuser-Grätzer Prize* celebrates

- The publication of the two volumes of
  Lattice Theory: Special Topics and Applications
  (G. Grätzer and F. Wehrung eds.) Springer Basel (Birkhäuser), 2014–2016;
- The 75th issue of the journal *Algebra Universalis* founded by Grätzer in 1971;
- Grätzer's 80th birthday.

The prize of $6,000 was awarded for the best research article on any of the topics in Grätzer's chapters in the first volume of *Special Topics*:

- planar semimodular lattices
- sectionally complemented lattices
- complete congruence lattices
- the order of principal congruences

The winner of the prize is Gábor Czédli (Bolyai Institute, University of Szeged), for his article on the third topic:

Complete congruence lattices of two related modular lattices.

This paper outlines in detail the background for this problem, for more detail, see my Chapter 10 in the first volume of *Special Topics.*

A series of papers

- K. Reuter and R. Wille 1987
- G. Grätzer 1988
- G. Grätzer 1989
- S.-K. Teo 1990
- G. Grätzer 1990
- G. Grätzer, H. Lakser, and B. Wolk 1991
- G. Grätzer and H. Lakser 1991
- R. Freese, G. Grätzer, and E. T. Schmidt 1991
- G. Grätzer and H. Lakser 1992
- G. Grätzer and E. T. Schmidt
  - 1993 (two papers)
  - 1995 (three papers)
  - 1997
  - 2001

culminated in two major results:

(i) Every complete lattice L can be represented as the lattice $\operatorname{Com} K$ of complete congruence relations of a complete, modular, 2-distributive, strongly atomic lattice K.
(ii) Every complete lattice L can be represented as the lattice $\operatorname{Com} K$ of complete congruence relations of a complete distributive lattice K.

Czédli's paper gives a simpler proof of the first result and a number of "two lattice" variants. (Part V of my book, *The Congruences of a Finite Lattice*, second edition, 2016, deals with "two lattice" variants for the finite case.) In his results, he takes two complete lattices K and K' that are interrelated, for instance, K is a complete sublattice of K'. Then the extension map (mapping a complete congruence $\boldsymbol{\alpha}$ of K to the smallest complete congruence $\boldsymbol{\alpha}'$ of K' containing $\boldsymbol{\alpha}$) has the following properties:

(1) it is 0-preserving;
(2) it is a $\bigvee$-homomorpism;
(3) it is 0-separating (only 0 maps to 0).

One of Czédli's results (modeled after G. Grätzer and H. Lakser 1986) proves the converse—we state it in a very simple, nontechnical form.

Theorem. *Let A and A' be complete lattices and let $f\colon A \to A'$ be a map satisfying* (1)–(3)*. Then there exist a complete, strongly atomic, modular lattice K' and a principal ideal K of K' such that* $\operatorname{Com} K$*,* $\operatorname{Com} K'$*, and the extension map represent A, A', and f.*

The proof is amazingly complicated with many interesting ideas and long computations. A very deserving recipient of the *Birkhäuser-Grätzer Prize.*

George Grätzer

Toronto, ON, Canada