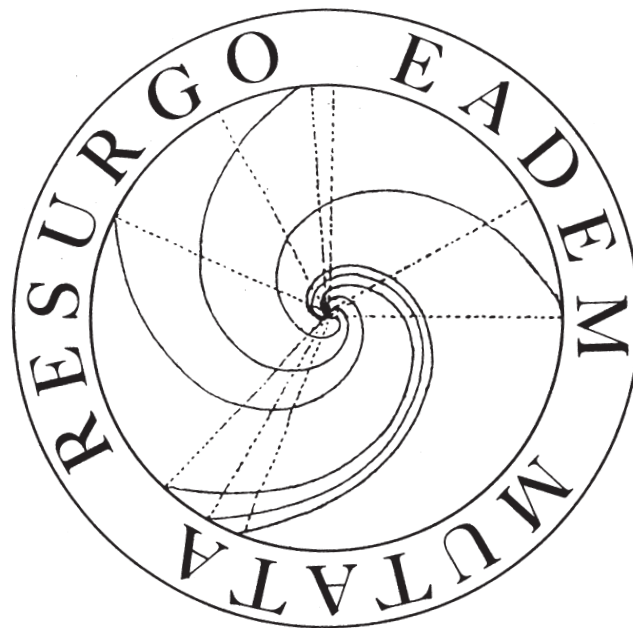


THE COLLECTED SCIENTIFIC PAPERS
OF THE MATHEMATICIANS AND PHYSICISTS
OF THE BERNOULLI FAMILY



Edited on behalf of
the Naturforschende Gesellschaft in Basel
and the Otto Spiess-Stiftung,
with support of the Schweizerischer Nationalfonds
and the Verein zur Förderung der Bernoulli-Edition

Birkhäuser



Birkhäuser Verlag AG
Basel · Boston · Berlin

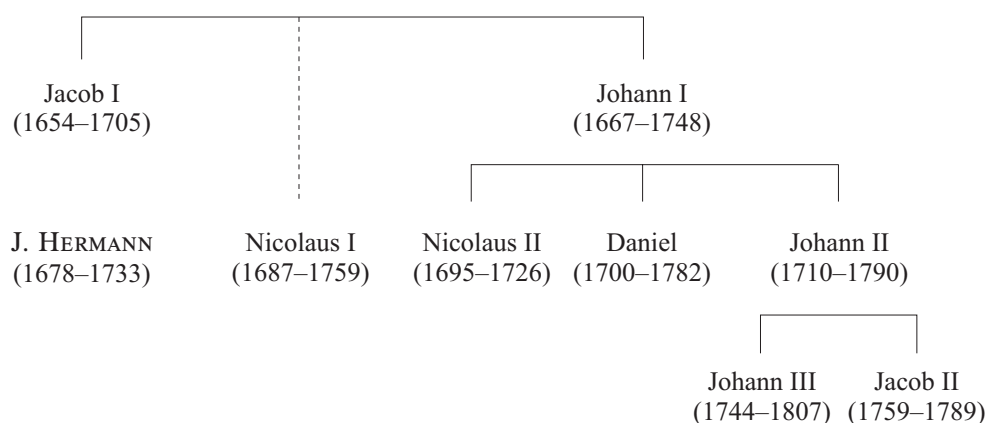
The Scientific Legacy of the Bernoullis

Modern science is predominantly based on the discoveries in the fields of mathematics and the natural sciences in the 17th and 18th centuries. Eight members of the Bernoulli family as well as the Bernoulli disciple Jacob Hermann made significant contributions to this development in the areas of mathematics, physics, engineering and medicine. Some of their most influential achievements may be listed as follows:

- Jacob Bernoulli's pioneering work in probability theory, which included the discovery of the Law of Large Numbers, the basic theorem underlying all statistical analysis
- Jacob's determination of the form taken by a loaded beam, the first attempt at a systematic formulation of elasticity theory
- the calculus of variations, invented by Jacob and by his brother Johann. Variational principles are the basis of much of modern physics
- the contributions of both Jacob and Johann to analysis, differential geometry and mechanics, which developed and disseminated Leibniz's calculus
- the formulation of Newtonian mechanics in the differential form by which we know it today, pioneered by Jacob Hermann and by Johann I Bernoulli
- Johann's work on hydrodynamics. Not well known until recently, it is now highly regarded by historians of physics
- Daniel Bernoulli's energy theorem for stationary flow, universally used in hydrodynamics and aerodynamics, and his derivation of Boyle's law, which for the first time explains macroscopic properties of gases by molecular motion, thus marking the beginning of kinetic gas theory
- the Bernoullis' contributions to the theory of oscillations, especially Daniel's discovery of the main theorems on stationary modes. Johann II considered, but rejected, a theory of transversal wave optics; Jacob II came tantalizingly close to formulating the equations for the vibrating plate – an important topic of the time
- the important steps Daniel Bernoulli took toward a theory of errors. His efforts to improve the apparatus for measuring the inclination of the Earth's magnetic field led him to the first systematic evaluation of experimental errors
- Daniel's achievements in medicine, including the first computation of the work done by the human heart. He also pioneered in applying statistical methods to mortality tables and advocated inoculation against smallpox.

Editorial Principles

Among the great editions of scientific works of the 17th and 18th centuries, the Bernoulli Edition occupies a special position; unlike, for example, the Leibniz and the Euler Edition, it is dedicated not to a single author, but to a group of nine closely related authors.



The Bernoulli Edition presents a critical, fully annotated edition of the collected works and correspondence of these authors. It includes their scientific publications, the most important of those texts which exist only in manuscript form, and an extensive selection of their correspondence, for the most part hitherto unpublished.

The Bernoulli Edition establishes an authoritative version of the source texts, starting from either the original publications or the manuscripts. The commentaries facilitate access to the historical texts for the modern reader by providing interpretative introductions, explanatory notes and indexes. All texts are printed in the original language (mainly Latin and French); the commentaries are mostly in English (in the past, some volumes have been annotated in French, German or Italian). Copious illustrations present figures from original printings as well as samples from holographs.

The Bernoulli Edition is divided into two sections, one for the Collected Works and one for the Correspondence. Each section is composed of several series, each of these being dedicated to one or, in some cases, to two of the nine authors. The following pages reflect this arrangement.

JACOB BERNOULLI



Jacob Bernoulli (1654–1705)

Jacob studied theology at Basel and at the same time taught himself mathematics. After a short period as a private tutor in Geneva and France, he travelled through the Netherlands, England and Germany. During this journey he became acquainted with the contemporary mathematics and physics of Descartes, Huygens, Wallis and Barrow. In 1687 Jacob was appointed to the chair of mathematics at the University of Basel. In the following years he rediscovered, together with his brother Johann, the secrets of Leibniz's calculus and began a correspondence with Leibniz. Jacob published his discoveries in more than sixty papers in the scientific journals of his time. For himself, he recorded his thoughts in a scientific diary (*Meditationes*), which is an outstanding testimony to his mathematical genius. Jacob published no books; his *Ars Conjectandi* remained unfinished and did not appear until eight years after his death. Recognized as one of the greatest mathematicians of his time, Jacob died in Basel in 1705.

DIE WERKE VON JAKOB BERNOULLI

Available:

Vol. 1:

Astronomie, Philosophia naturalis

Edited by J. O. Fleckenstein

1969 (2nd printing 1989).

541 pages. 224 ills. Hardcover

ISBN 3-7643-0028-0

Vol. 2:

Elementarmathematik

Edited by C. S. Roero and T. Viola

1989. 698 pages. 291 ills. Hardcover

ISBN 3-7643-1891-0

Vol. 3:

Wahrscheinlichkeitsrechnung

Edited by B. L. van der Waerden

1975. 594 pages. 34 ills. Hardcover

ISBN 3-7643-0713-7

Vol. 4:

Reihentheorie

Edited by A. Weil

1993. 320 pages. 40 ills. Hardcover

ISBN 3-7643-2453-8

"The devices of this calculus are yet known to few people, and I do not know anybody who has understood my meaning better than this famous man."

Leibniz on Jacob Bernoulli, 1690

New:

Vol. 5:

Differentialgeometrie

Edited by A. Weil and M. Mattmüller
1999. 472 pages. 186 ill. Hardcover
ISBN 3-7643-5779-7

This volume contains Jacob Bernoulli's contributions to differential geometry, a field marked by some of his greatest achievements. Between 1690 and 1700, Bernoulli published twelve treatises in the scientific journal *Acta Eruditorum* on the use of infinitesimal methods to explore geometrical questions; preparatory notes for most of these papers and on many other themes from his scientific diary *Meditationes* are published here for the first time.

Among the curves considered are the isochrones (lines of constant descent), the parabolic spiral, the loxodrome, the cycloid, the tractrix and the logarithmic spiral (Bernoulli's *spira mirabilis*, which also adorns his tombstone). The description of these curves by differential equations and by geometrical constructions, their rectification and quadrature, and the determination of their evolutes and caustics offered Bernoulli a range of challenging problems, many of them relevant to mechanical or optical applications.

Together with Huygens, Tschirnhaus and L'Hôpital, Jacob Bernoulli and his younger brother Johann belonged to the small group of scientists who discussed and developed the cryptic ideas first hinted at by Leibniz in the 1680s. Through the intense, often competitive intellectual exchange pursued in their publications and correspondence, differential calculus gradually evolved into a powerful analytical method.

In preparation:

Vol. 6:

Mechanik

Edited by P. Radelet-de Grave and
D. Speiser

The first part of this volume contains Jacob Bernoulli's long and eventually successful struggle to derive the theory of the physical pendulum not from the energy theorem – as Huygens had done –, but from the principle of the moment of forces (the "Law of the Lever" as he calls it). In the *Meditationes* one finds Bernoulli's ideas concerning basic concepts and laws, such as the parallelogram of forces.

The second part of the volume contains the theory of elastic beams, including the case of the initially curved beam. Here Bernoulli balances the moment of the "applied" force against the restitutive moment induced in each fiber of the beam. He then uses his formula for the radius of curvature, his famous "*theorema aureum*". Because of his experiments, Bernoulli was fully aware that for each material he needed a specific constitutive equation linking the curvature at each point of the beam to the tension.

Also available:**DER BRIEFWECHSEL VON
JACOB BERNOULLI**

Edited by A. Weil,
with contributions by C. Truesdell and
F. Nagel
1993. 328 pages. 36 ill. Hardcover
ISBN 3-7643-2950-5

JOHANN I BERNOULLI



Johann I Bernoulli (1667–1748)

Johann studied medicine at Basel and was introduced to mathematics by his elder brother Jacob. Both brothers successfully taught themselves Leibniz's calculus and forged it into a powerful tool for solving several of the most difficult mathematical problems of their time. As a result of his private lectures with Johann, the Marquis de l'Hôpital was encouraged to write the first textbook on calculus, the *Analyse des infiniment petits* (1696). In 1695, Johann Bernoulli was appointed to the chair of mathematics at Groningen; when his brother Jacob died in 1705, Johann became his successor at Basel. From there he disseminated the methods and results of his research through teaching, publishing and corresponding with mathematicians and scientists throughout Europe. Among his disciples he could count, in addition to three of his sons, Maupertuis, Cramer, Clairaut and, most notably, Euler. Johann Bernoulli was also involved in most of the public scientific controversies of the time; among his adversaries we find his brother Jacob and his son Daniel, the English school of Newton's followers as well as

some of Johann's own former disciples. Acknowledged after the deaths of Leibniz and Newton as the foremost mathematician of his time, Johann Bernoulli died in Basel in 1748.

DIE WERKE VON JOHANN I UND NICOLAUS II BERNOULLI

This series is currently being planned. It will contain the following ten volumes:

- 1 Mathematik I
(Algebra; Analysis: Series, Integrals, Differential Equations; Probability)
- 2 Mathematik II
(Geometry)
- 3 Mathematik III
(Calculus: Lectures to L'Hôpital)
- 4/5 Mathematik IV
(Trajectories, Priority Dispute)
- 6 Mechanik I
(General Mechanics, Elasticity)
- 7 Mechanik II
(Friction Problems, Hydrodynamics)
- 8 Physik I
(Technology, Natural Philosophy)
- 9 Physik II
(General Physics, Medicine and Physiology, Varia)
- 10 Weitere Schriften
(Biographical and Academic Items, School Reform, Reading Notes)

*"Son esprit vit la vérité,
Et son cœur connut la justice,
Il a fait l'honneur de la Suisse
Et celui de l'humanité."*

Voltaire on Johann I Bernoulli, 1742

In preparation:

Vol. 6:

Mechanik I

Edited by P. Villaggio

This volume – which will be the first to appear in the series of Johann I Bernoulli's works – contains contributions ranging from differential statics and point mechanics to problems of elasticity.

To the first group belongs Bernoulli's discovery of the form of the catenary, which brought him immediate recognition among a circle of scientists in Paris. In the second group we find his extension of Jacob Hermann's solution to the direct Kepler problem (starting from the equation of motion rather than from the orbit, as Newton had). Johann Bernoulli's solution is essentially the one still presented in textbooks today; it contains the first complete discussion of the various integration constants, thus opening the path to analytical mechanics. Bernoulli's contributions to elasticity, especially to the theory of loaded strings and the multiple pendulum, culminate in a paper – written at the age of seventy-five! – where Bernoulli comes very close to the general equations of motion of a continuum.

Available:

**DIE STREITSCHRIFTEN VON
JACOB UND JOHANN BERNOULLI**

Variationsrechnung

Edited by H. H. Goldstine

1991. 630 pages. 180 ills. Hardcover

ISBN 3-7643-2348-5

**DER BRIEFWECHSEL VON
JOHANN I BERNOULLI**

Available:

Vol. 1:

**Der Briefwechsel mit Jakob Bernoulli,
dem Marquis de l'Hôpital u.a.**

Edited by O. Spiess

1955. 531 pages. 128 ills. Hardcover

ISBN 3-7643-0027-2

Vol. 2:

**Der Briefwechsel mit Pierre Varignon,
Teil I: 1692–1702**

Edited by P. Costabel and J. Peiffer

1988. 460 pages. 77 ills. Hardcover

ISBN 3-7643-1183-5

Vol. 3:

**Der Briefwechsel mit Pierre Varignon,
Teil II: 1703–1714**

Edited by P. Costabel and J. Peiffer

1992. 648 pages. 74 ills. Hardcover

ISBN 3-7643-2637-9

In preparation:

Vol. 4:

**Der Briefwechsel mit Pierre Varignon,
Teil III: 1715–1722**

Edited by J. Peiffer



DANIEL BERNOULLI



Daniel Bernoulli (1700–1782)

Daniel, the second son of Johann I Bernoulli, was born in Groningen. He studied medicine at Basel, Heidelberg and Strasbourg. Continuing his education in Venice, he became increasingly involved in mathematics. In 1725, Daniel was appointed to the Imperial Academy of St. Petersburg, together with his brother Nicolaus II. There he turned his attention to mechanics and wrote the first draft of his *Hydrodynamica*; the final version, his most important single work, was to be published in Strasbourg in 1738. In 1733, Daniel returned to Basel, at first to the chair of anatomy and botanics; in 1743 he became professor of medicine, and in 1750 he was finally appointed to the chair of physics. In more than seventy papers published in various journals, Daniel treated problems in physics and mathematics, some of them involving applications of probability theory. He was awarded the prize of the Paris *Académie des sciences* ten times. Daniel Bernoulli died in Basel in 1782.

DIE WERKE VON DANIEL BERNOULLI

Available:

Vol. 1:

Medizin und Physiologie

Edited by V. Zimmermann

Mathematische Jugendschriften

Edited by U. Bottazzini,

with a contribution by G. K. Mikhailov

Positionsastronomie

Edited by M. Howald-Haller

1996. 528 pages. 81 ills. Hardcover

ISBN 3-7643-5272-8

Vol. 2:

Analysis, Wahrscheinlichkeitsrechnung

Edited by L. P. Bouckaert

and B. L. van der Waerden

1982. 403 pages. 12 ills. Hardcover

ISBN 3-7643-1084-7

Vol. 3:

Mechanik

Edited by D. Speiser,

A. de Baenst-Vandenbroucke,

J. L. Pietenpol and P. Radelet-de Grave

1987. 484 pages. 115 ills. Hardcover

ISBN 3-7643-1213-0

Vol. 7:

Magnetismus

Edited by P. Radelet-de Grave,

with a contribution on electricity

by D. Speiser

Technologie I

Edited by A. Englebert

1994. 357 pages. 90 ills. Hardcover

ISBN 3-7643-2808-8

To appear:

Vol. 5:

Hydrodynamik II

Edited by G. K. Mikhailov

2001. Approx. 750 pages. 142 ills.

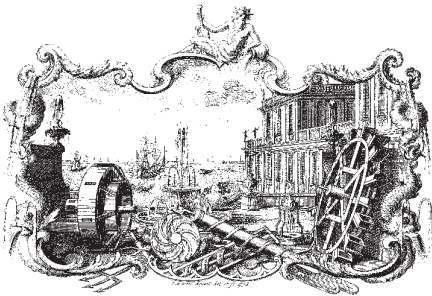
Hardcover

ISBN 3-7643-6260-X

This volume contains Daniel Bernoulli's most important work by far, the justly famous *Hydrodynamica*. The new treatment of the flow of fluids presented here is almost entirely based on the energy principle and on the "principle of planar sections" that characterizes the incompressibility of a liquid. From these principles Bernoulli derives a host of results and discusses very carefully a series of experiments and technical applications.

Today the book is best known for the derivation of the law of Towneley, Boyle and Mariotte from an atomistic hypothesis with the help of a probabilistic computation. Many years later, this train of thought gave rise to statistical mechanics.

Another result still famous is the so-called Bernoulli equation, which links the pressure of a stationary flow to its velocity, thus extending the established science of hydraulics into the more comprehensive hydrodynamics.



In preparation:

Vol. 4:

Hydrodynamik I

Edited by G. K. Mikhailov

Vol. 6:

Elastizität

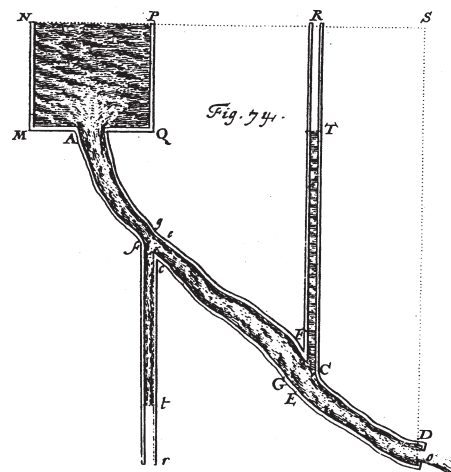
Edited by D. Speiser

and P. Radelet-de Grave

Vol. 8:

Technologie II

Edited by F. Cerulus



"Nobody found more expedients in analysis for submitting to calculation all circumstances of a phenomenon; nobody knew better how to arrange an experiment, in order to confirm theoretical results or to give a basis for calculation. Everywhere he is a philosopher and a physicist as well as a mathematician."

Condorcet on Daniel Bernoulli, 1782

Critical Acclaim for the Bernoulli Edition

Hopefully the still unraised treasure of the Bernoullis' correspondences and writings will soon be made accessible in further volumes. The section on hand, exemplary by its structure, by the presentation of the source text and by the scope and reliability of its indexes, is a masterly editorial accomplishment.

Deutsche Literaturzeitung 1956

All in all, the edition, which has started excellently, is indispensable for the expert studying the early history of higher analysis; but its characteristics also render it highly interesting for any professional mathematician with historical inclinations.

J. E. Hofmann, Jahresbericht DMV 1957

Both the editors and the publishers are to be congratulated on producing a volume of meticulously researched detail and highly perceptive interpretations, which is at the same time quite exceptional, certainly for a scientific work, in the aesthetic quality of its design and production.

E. Aiton, Göttingische Gelehrte Anzeigen 1990

The Bernoulli Edition is opening new horizons for the history of science and technology in the 18th century, and it is to be hoped that more volumes, edited by equally competent commentators, will shortly follow.

A. Kleinert, NTM 1997

The Second Stage: Plans for the remaining Collected Works Series

While bringing the series which contain the works of Jacob I and of Daniel Bernoulli (as documented on the last pages) to a close, the Bernoulli Edition is preparing the publication of the seven other authors' works. The first priority lies with the series comprising the works of Johann I Bernoulli (as well as those of his eldest son Nicolaus II), for which the plan is presented above. The remaining series of Collected Works are currently being planned as follows:

Jacob Hermann / Nicolaus I Bernoulli, Werke (4 vols.)

- 1 Mathematik
(Analysis, Geometry, Probability)
- 2/3 Mechanik I
(J. Hermann, *Phoronomia*)
- 4 Mechanik II, Physik
(Central Forces, Optics, Elasticity, Hydrodynamics, Varia)

Johann II Bernoulli, Werke (1 vol.)

Jacob II Bernoulli, Werke (1 vol.)

Johann III Bernoulli, Werke

Several volumes of these series have been assigned to commentators; work on some of them is in progress, and indeed the first volume of the second stage (vol. 6 of Johann Bernoulli's Works) has been deposited with the Bernoulli Edition.

The Third Stage: Plans for the Correspondence Series

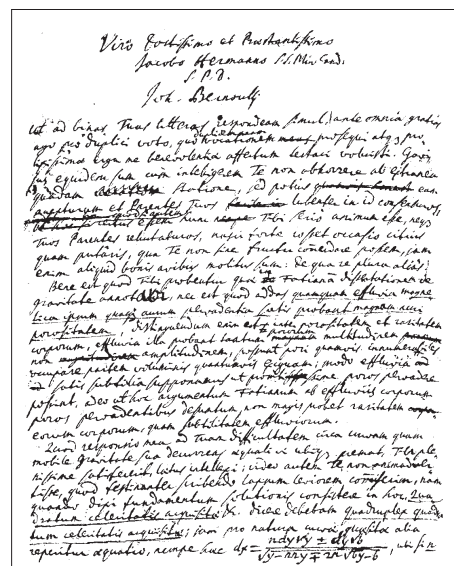
The correspondence of the mathematicians, physicists and physicians of the Bernoulli family and of Jacob Hermann comprises about 8000 preserved letters exchanged with more than 500 correspondents covering a time span of over 130 years. Among the Bernoullis' correspondents one finds almost all distinguished members of the scientific community of that period: Euler, Fontenelle, Goldbach, Leibniz, L'Hôpital, Maupertuis, Michelotti, de Moivre, Montmort, Newton, Riccati, Varignon, Voltaire, Wolff and many others.

It has always been a major concern of the Bernoulli Edition to bring this unique historical treasure to light and to make it accessible to the public. In addition to the four volumes already published, the Edition archive contains typewritten transcriptions of nearly 3800 letters including a name index worked out by Otto Spiess. Moreover, a database containing all existing letters has recently been established by Fritz Nagel.

In view of its extent and of the constraints on the resources of the Bernoulli Edition, a publication of the complete Bernoulli correspondence does not appear feasible. Accordingly it has been necessary to make a selection of letter exchanges worth being edited. The criteria of this selection were the scientific content of the letters and their importance for our knowledge of the Bernoulli circle and for the history of science in general. A report on the result of this procedure, authored by Fritz Nagel and David Speiser, has been presented to the

scientific advisory board and is available at the Bernoulli edition. Its main points are as follows: In order to enable the reader to follow the dialogue of the partners, each letter exchange – if edited – will be integrally published. All letters not published in the Bernoulli Edition will be listed and documented by giving a formal description (date, place, shelf mark) and a short summary of their contents. Those summaries are currently being drawn up by Fritz Nagel within the framework of a research project sponsored by the Swiss National Foundation.

About 140 correspondences containing 1700 letters are to be edited; we estimate this selection to make up 15 volumes. Taking into account some special cases, for which the Bernoulli Edition hopes to establish an international collaboration (e.g., the correspondences with Dortous de Mairan, La Condamine and Michelotti), the edition of the Bernoulli correspondences would necessitate 25 volumes altogether.



THE BERNOULLI EDITION: COLLABORATORS AND ADDRESSES

General Editor:

D. Speiser
Bromhübelweg 5
CH-4144 Arlesheim

Forschungsstelle Basel: (Correspondence Series)

F. Nagel
Universitätsbibliothek
Schönbeinstr. 18/20
CH-4056 Basel
Tel. ++41 / 61 / 267 31 41
Fax ++41 / 61 / 267 31 03
e-mail: bernoullied@ubaclu.unibas.ch

Unité de Recherches Louvain: (Collected Works Series)

P. Radelet-de Grave
Institut de Physique Théorique
2, chemin du Cyclotron
B-1348 Louvain-la-Neuve
Tel. ++32 / 10 / 47 32 80
Fax ++32 / 10 / 47 24 14
e-mail: radelet@fyoma.ucl.ac.be

Secretary:

M. Mattmüller, Basel

Scientific Contributors and Editors:

E. Benvenuto †, Genova	G. K. Mikhailov, Moskva
M. Blay, Paris	F. Nagel, Basel
U. Bottazzini, Milano	D. Ó Mathúna, Dublin
L. Bouckaert †, Leuven	J. Peiffer, Paris
P. Costabel †, Paris	J. L. Pietenpol, Raleigh
F. Cerulus, Leuven	P. Radelet-de Grave, Louvain-la-Neuve
A. de Baenst-Vandenbroucke, Namur	C. S. Roero, Torino
J. Dhombres, Nantes	D. Speiser, Basel
A. Englebert, Bruxelles	O. Spiess †, Basel
J. O. Fleckenstein †, Basel	H. Straub †, Basel
E. Giusti, Firenze	C. Truesdell, Baltimore
H. H. Goldstine, Philadelphia	B. L. van der Waerden †, Zürich
H.-J. Heß, Hannover	T. Viola †, Torino
M. Howald-Haller, Basel	P. Villaggio, Pisa
A. Lederer †, Louvain	A. Weil †, Princeton
G. Maltese, Roma	V. Zimmermann, Göttingen
M. Mattmüller, Basel	

Birkhäuser



Birkhäuser Verlag AG
Basel · Boston · Berlin