

# The Bernoulli Edition

The Collected  
Scientific Papers of  
the Mathematicians  
and Physicists of the  
Bernoulli Family



Edited on behalf of the *Naturforschende Gesellschaft in Basel* and the *Otto Spieß-Stiftung*, with support of the *Schweizerischer Nationalfonds* and the *Verein zur Förderung der Bernoulli-Edition*

*Birkhäuser*



# The Scientific Legacy of the Bernoullis

Modern science is predominantly based on the discoveries in the fields of mathematics and the natural sciences in the 17th and 18th centuries. Eight members of the Bernoulli family as well as the Bernoulli disciple Jacob Hermann made significant contributions to this development in the areas of mathematics, physics, engineering and medicine. Some of their most influential achievements may be listed as follows:

- ▲ Jacob Bernoulli's pioneering work in probability theory, which included the discovery of the Law of Large Numbers, the basic theorem underlying all statistical analysis
- ▲ Jacob's determination of the form taken by a loaded beam, the first attempt at a systematic formulation of elasticity theory
- ▲ the calculus of variations, invented by Jacob and by his brother Johann. Variational principles are the basis of much of modern physics
- ▲ the contributions of both Jacob and Johann to analysis, differential geometry and mechanics, which developed and disseminated Leibniz's calculus
- ▲ the formulation of Newtonian mechanics in the differential form by which we know it today, pioneered by Jacob Hermann and by Johann I Bernoulli
- ▲ Johann's work on hydrodynamics. Not well known until recently, it is now highly regarded by historians of physics
- ▲ Daniel Bernoulli's energy theorem for stationary flow, universally used in hydrodynamics and aerodynamics, and his derivation of Boyle's law, which for the first time explains macroscopic properties of gases by molecular motion, thus marking the beginning of kinetic gas theory

- ▲ the Bernoullis' contributions to the theory of oscillations, especially Daniel's discovery of the main theorems on stationary modes. Johann II considered, but rejected, a theory of transversal wave optics; Jacob II came tantalizingly close to formulating the equations for the vibrating plate – an important topic of the time
- ▲ the important steps Daniel Bernoulli took toward a theory of errors. His efforts to improve the apparatus for measuring the inclination of the Earth's magnetic field led him to the first systematic evaluation of experimental errors
- ▲ Daniel's achievements in medicine, including the first computation of the work done by the human heart. He also pioneered in applying statistical methods to mortality tables and advocated inoculation against smallpox.

## Editorial Matters

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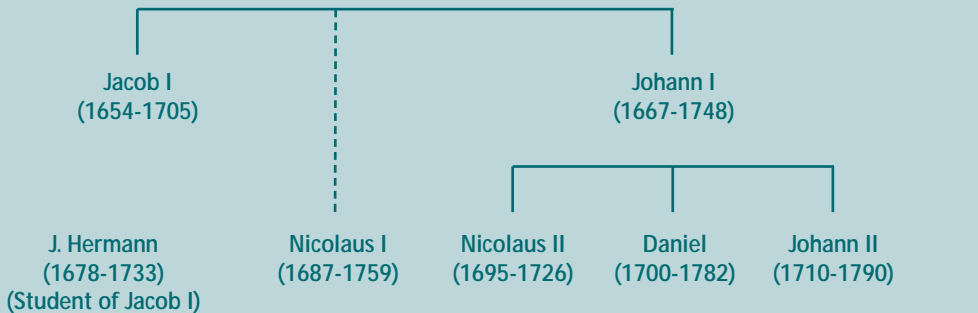
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# Editorial Principles

Among the great editions of scientific works of the 17th and 18th centuries, the Bernoulli Edition occupies a special position; unlike, for example, the Leibniz and the Euler Edition, it is dedicated not to a single author, but to a group of nine closely related authors.



The Bernoulli Edition presents a critical, fully annotated edition of the collected works and correspondence of these authors. It includes their scientific publications, the most important of those texts which exist only in manuscript form, and an extensive selection of their correspondence, for the most part hitherto unpublished. The Bernoulli Edition establishes an authoritative version of the source texts, starting from either the original publications or the manuscripts. The commentaries facilitate access to the historical texts for the modern reader by providing interpretative introductions, explanatory notes and indexes. All texts are printed in the original language (mainly Latin and French); the commentaries are mostly in English (in the past, some volumes have been annotated in French, German or Italian). Copious illustrations present figures from original printings as well as samples from holographs. The Bernoulli Edition is divided into two sections, one for the Collected Works and one for the Correspondence. Each section is composed of several series, each of these being dedicated to one or, in some cases, to two of the nine authors. The following pages reflect this arrangement.





## Jacob Bernoulli (1654-1705)

Jacob studied theology in Basel and at the same time taught himself mathematics. After a short period as a private tutor in Geneva and France, he travelled through the Netherlands, England and Germany. During this journey he became acquainted with the contemporary mathematics and physics of Descartes, Huygens, Wallis and Barrow. In 1687 Jacob was appointed to the chair of mathematics at the University of Basel. In the following years he rediscovered, together with his brother Johann, the secrets of Leibniz's calculus and began a correspondence with Leibniz. Jacob published his discoveries in more than sixty papers in the scientific journals of his time. For himself, he recorded his thoughts in a scientific diary (*Meditationes*), which is an outstanding testimony to his mathematical genius. Jacob published no books; his *Ars Conjectandi* remained unfinished and did not appear until eight years after his death. Recognized as one of the greatest mathematicians of his time, Jacob died in Basel in 1705.

## Die Werke von Jacob Bernoulli

Available:

Vol. 2:

### Elementarmathematik

Edited by C. S. Roero and T. Viola  
1989. (2nd corrected printing, due in 2005)  
698 pages. 291 ills. Hardcover  
ISBN 3-7643-1891-0

Vol. 3:

### Wahrscheinlichkeitsrechnung

Edited by B. L. van der Waerden  
1975. 594 pages. 34 ills. Hardcover  
ISBN 3-7643-0713-7

Vol. 4:

### Reihentheorie

Edited by A. Weil  
1993. 320 pages. 40 ills. Hardcover

Teil 1: Reihentheorie  
ISBN 3-7643-2453-8

Teil 2: Briefwechsel  
ISBN 3-7643-2950-5

Teil 1 + 2  
ISBN 3-7643-2961-0

Vol. 5:

### Differentialgeometrie

Edited by A. Weil and M. Mattmüller  
1999. 472 pages. 186 ills. Hardcover  
ISBN 3-7643-5779-7

In preparation:

Vol. 1:

## Astronomie, Philosophia naturalis

2nd revised edition

Vol. 6:

## Mechanik

Edited by P. Radelet-de Grave and D. Speiser

The first part of this volume contains Jacob Bernoulli's long and eventually successful struggle to derive the theory of the physical pendulum not from the energy theorem - as Huygens had done -, but from the principle of the moment of forces (the "Law of the Lever" as he calls it). In the *Meditationes* one finds Bernoulli's ideas concerning basic concepts and laws, such as the parallelogram of forces.

The second part of the volume contains the theory of elastic beams, including the case of the initially curved beam. Here Bernoulli balances the moment of the "applied" force against the restitutive moment induced in each fiber of the beam. He then uses his formula for the radius of curvature, his famous *theorema aureum*. Because of his experiments, Bernoulli was fully aware that for each material he needed a specific constitutive equation linking the curvature at each point of the beam to the tension.

Also available:

## Der Briefwechsel von Jacob Bernoulli

Edited by A. Weil, with contributions by C. Truesdell and F. Nagel

1993. 328 pages. 36 ill. Hardcover

ISBN 3-7643-2950-5

*"The devices of this calculus are yet known to few people, and I do not know anybody who has understood my meaning better than this famous man."*

Leibniz on Jacob Bernoulli, 1690





## Johann I Bernoulli (1667-1748)

Johann studied medicine in Basel and was introduced to mathematics by his elder brother Jacob. Both brothers successfully taught themselves Leibniz's calculus and forged it into a powerful tool for solving several of the most difficult mathematical problems of their time. As a result of his private lectures with Johann, the Marquis de l'Hôpital was encouraged to write the first textbook on calculus, the *Analyse des infiniment petits* (1696). In 1695, Johann Bernoulli was appointed to the chair of mathematics at Groningen; when his brother Jacob died in 1705, Johann became his successor in Basel. From there he disseminated the methods and results of his research through teaching, publishing and corresponding with mathematicians and scientists throughout Europe. Among his disciples he could count, in addition to three of his sons, Maupertuis, Cramer, Clairaut and, most notably, Euler. Johann Bernoulli was also involved in most of the public scientific controversies of the time; among his adversaries we find his brother Jacob and his son Daniel, the English school of Newton's followers as well as some of Johann's own former disciples. Acknowledged after the deaths of Leibniz and Newton as the foremost mathematician of his time, Johann Bernoulli died in Basel in 1748.

## Die Werke von Johann I und Nicolaus II Bernoulli

This series is currently being planned. It will contain the following ten volumes:

### 1 Mathematik I

Algebra; Analysis: Series, Integrals, Differential Equations; Probability

### 2 Mathematik II

Geometry

### 3 Mathematik III

Calculus: Lectures to L'Hôpital

### 4/5 Mathematik IV

Trajectories, Priority Dispute

### 6 Mechanik I

General Mechanics, Elasticity

### 7 Mechanik II

Friction Problems, Hydrodynamics

### 8 Physik I

Technology, Natural Philosophy

### 9 Physik II

General Physics, Medicine and Physiology, Varia

### 10 Weitere Schriften

Biographical and Academical Items, School Reform, Reading Notes

*"Son esprit vit la vérité,  
Et son cœur connu la justice,  
Il a fait l'honneur de la Suisse  
Et celui de l'humanité."*

Voltaire on Johann I Bernoulli, 1742

In preparation:

Vol. 6:

## Mechanik I

Edited by P. Villaggio

This volume on mechanics of rigid and elastic bodies contains early papers concerning geometric statics, accompanied by works dealing with the motion of compound pendula and the deformation of beams. The papers on mechanics in this volume do not encompass the area of hydraulics, which occupies approximately one half of the papers dealing with mechanical problems and which are included in volume 7. This collection constitutes, roughly, one eighth of the entire work written by Bernoulli.

Vol. 2:

## Mathematik II (Geometry)

Edited by S. Roero and E. Giusti

This volume collects a large number of unpublished papers on the rectification of curves, the problem of the transformation of algebraic curves into other algebraic curves of the same length, the reptorial motion, spherical epicycloids, and geodesic lines. In letters and works we can see the analytical and geometrical approaches to these topics and the lively discussions entertained with illustrious contemporaries, like Leibniz, Tschirnhaus, Craig, Hermann, Burnet, de Moivre, and Grandi. The particularly interesting and rich manuscript contains 120 propositions and presents the variety of Johann Bernoulli's mathematical research from the period of his early youth to his late maturity. It allows us to discover the evolution of his thinking and his powerful mathematical techniques, as well as the influences received and exerted on 18th century mathematics.

## Die Streitschriften von Jacob und Johann Bernoulli

Available:

### Variationsrechnung

Edited by H. H. Goldstine  
1991. 630 pages. 180 ills. Hardcover  
ISBN 3-7643-2348-5

### Der Briefwechsel von Johann I Bernoulli

Available:

Vol. 1:

### Der Briefwechsel mit Jacob Bernoulli, dem Marquis de l'Hôpital u.a.

Edited by O. Spiess  
1955. 531 pages. 128 ills. Hardcover  
ISBN 3-7643-0027-2

Vol. 2:

### Der Briefwechsel mit Pierre Varignon

#### Teil I: 1692-1702

Edited by P. Costabel and J. Peiffer  
1988. 460 pages. 77 ills. Hardcover  
ISBN 3-7643-1183-5

Vol. 3:

### Der Briefwechsel mit Pierre Varignon

#### Teil II: 1703-1714

Edited by P. Costabel and J. Peiffer  
1992. 648 pages. 74 ills. Hardcover  
ISBN 3-7643-2637-9

In preparation:

Vol. 4:

### Der Briefwechsel mit Pierre Varignon

#### Teil III: 1715-1722

Edited by J. Peiffer



## Daniel Bernoulli (1700-1782)

Daniel, the second son of Johann I Bernoulli, was born in Groningen. He studied medicine in Basel, Heidelberg and Strasbourg. Continuing his education in Venice, he became increasingly involved in mathematics. In 1725, Daniel was appointed to the Imperial Academy of St. Petersburg, together with his brother Nicolaus II. There he turned his attention to mechanics and wrote the first draft of his *Hydrodynamica*; the final version, his most important single work, was to be published in Strasbourg in 1738. In 1733, Daniel returned to Basel, at first to the chair of anatomy and botany; in 1743 he became professor of medicine, and in 1750 he was finally appointed to the chair of physics. In more than seventy papers published in various journals, Daniel treated problems in physics and mathematics, some of them involving applications of probability theory. He was awarded the prize of the Paris *Académie des sciences* ten times. Daniel Bernoulli died in Basel in 1782.



## Die Werke von Daniel Bernoulli

Available:

Vol. 1:

### Medizin und Physiologie

Edited by V. Zimmermann

### Mathematische Jugendschriften

Edited by U. Bottazzini,  
with a contribution by G. K. Mikhailov

### Positionsastronomie

Edited by M. Howald-Haller  
1996. 528 pages. 81 ills. Hardcover  
ISBN 3-7643-5272-8

Vol. 2:

### Analysis, Wahrscheinlichkeitsrechnung

Edited by L. P. Bouckaert and B. L. van der Waerden  
1982. 403 pages. 12 ills. Hardcover  
ISBN 3-7643-1084-7

Vol. 3:

### Mechanik

Edited by D. Speiser, A. de Baenst-Vandenbroucke,  
J. L. Pietenpol and P. Radelet-de Grave  
1987. 484 pages. 115 ills. Hardcover  
ISBN 3-7643-1213-0

Vol. 5:

### Hydrodynamik II

Edited by G. K. Mikhailov  
2002. 756 pages. 142 ills. Hardcover  
ISBN 3-7643-6260-X

Volume 4 (Hydrodynamik I, in preparation) is devoted to the early works of Daniel Bernoulli on hydraulics preceding his famous *Hydrodynamik* that is published in Vol. 5 (*Hydrodynamik II*). In these works Bernoulli gave the first theoretical foundations of hydraulics and investigated the motion of bodies in resisting media. Volume 4 will also contain the previously unpublished manuscript version of Bernoulli's *Hydrodynamik* that shows



the formation of this epoch-making work. A paper on the principles of hydraulics written by Leonhard Euler simultaneously to Bernoulli's work, first discovered in the middle of the 20th century, is given in the volume as an appendix. The original texts are prefaced by an essay on the development of theoretical hydraulics in the second half of the 1720s and in the beginning of the 1730s.

Vol. 7:

## Magnetismus

Edited by P. Radelet-de Grave, with a contribution on electricity by D. Speiser

## Technologie I

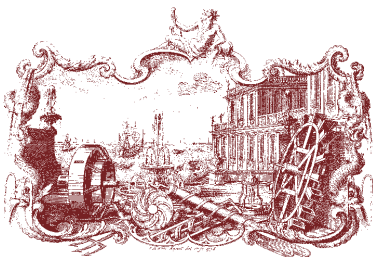
Edited by A. Englebert  
1994. 357 pages. 90 ill. Hardcover  
ISBN 3-7643-2808-8

Vol. 8:

## Technologie II

Edited by F. Cerulus  
2004. 380 pages. Hardcover  
ISBN 3-7643-7007-6

This volume consists mainly of three longer articles that have been awarded by the Paris academy of science and deal with nautical mathematical problems. The first article concentrates on ship propulsion, ranging from the rudder up to jet propulsion making first intensive use of the terms power and efficiency factor. In the second article Bernoulli deals with ship oscillations; he is the first who establishes a theory for ship movement in rough sea. The third article analyses the movement of the anchor on the seafloor. Here Bernoulli uses the chainline to accentuate the role of the cable. These three articles are embraced by two works on astronomy and another one about spiral pumps.



In preparation:

Vol. 4:

## Hydrodynamik I

Edited by G. K. Mikhailov

Vol. 6:

## Elastizität

Edited by D. Speiser and P. Radelet-de Grave

*"Nobody found more expedients in analysis for submitting to calculation all circumstances of a phenomenon; nobody knew better how to arrange an experiment, in order to confirm theoretical results or to give a basis for calculation. Everywhere he is a philosopher and a physicist as well as a mathematician."*

Condorcet on Daniel Bernoulli, 1782



## Critical Acclaim for the Bernoulli Edition

*"Hopefully the still unraised treasure of the Bernoullis' correspondences and writings will soon be made accessible in further volumes. The section on hand, exemplary by its structure, by the presentation of the source text and by the scope and reliability of its indexes, is a masterly editorial accomplishment."*

Deutsche Literaturzeitung 1956

*"All in all, the edition, which has started excellently, is indispensable for the expert studying the early history of higher analysis; but its characteristics also render it highly interesting for any professional mathematician with historical inclinations."*

J. E. Hofmann, Jahresbericht DMV 1957

*"Both the editors and the publishers are to be congratulated on producing a volume of meticulously researched detail and highly perceptive interpretations, which is at the same time quite exceptional, certainly for a scientific work, in the aesthetic quality of its design and production."*

E. Aiton, Göttingische Gelehrte Anzeigen 1990

*"The Bernoulli Edition is opening new horizons for the history of science and technology in the 18th century, and it is to be hoped that more volumes, edited by equally competent commentators, will shortly follow."*

A. Kleinert, NTM 1997

## Plans for the remaining Collected Works Series

### Jacob Hermann / Nicolaus I Bernoulli, Werke (4 vols)

#### 1 Mathematik

Analysis, Geometry, Probability

#### 2/3 Mechanik I

J. Hermann, Phoronomia

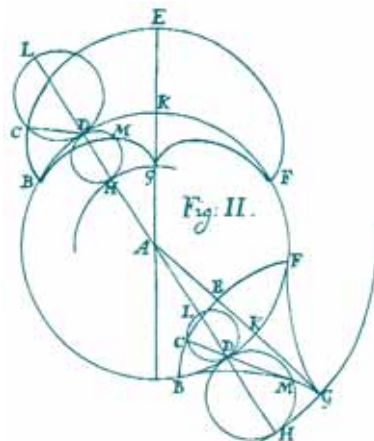
#### 4 Mechanik II, Physik

Central Forces, Optics, Elasticity, Hydrodynamics, Varia

### Johann II Bernoulli, Werke (1 vol.)

### Jacob II Bernoulli, Werke (1 vol.)

### Johann III Bernoulli, Werke



## Plans for the Correspondence Series

The correspondence of the mathematicians, physicists and physicians of the Bernoulli family and of Jacob Hermann comprises about 8000 preserved letters exchanged with more than 500 correspondents covering a time span of over 130 years. Among the Bernoullis' correspondents one finds almost all distinguished members of the scientific community of that period: Euler, Fontenelle, Goldbach, Leibniz, L'Hôpital, Maupertuis, Michelotti, de Moivre, Montmort, Newton, Riccati, Varignon, Voltaire, Wolff and many others.

It has always been a major concern of the Bernoulli Edition to bring this unique historical treasure to light and to make it accessible to the public. In addition to the four volumes already published, the Edition archive contains typewritten transcriptions of nearly 3800 letters including a name index worked out by Otto Spiess. Moreover, a database containing all existing letters has recently been established by Fritz Nagel. In view of its extent and of the constraints on the resources of the Bernoulli Edition, a publication of the complete Bernoulli correspondence does not appear feasible.

Accordingly it was decided to put the text of all the letters onto the World Wide Web in order to make them accessible to the scientific editors exclusively, and later, when the texts will be well established, to everybody. Within these numerous letters a selection has to be done of exchange of letters worth being edited on paper. A first proposition has been done by Fritz Nagel and David Speiser, and was presented to the scientific advisory board. The criteria of their selection were the scientific content of the letters and their importance for our knowledge of the Bernoulli circle and for the history of science in general. This proposition organizes the volumes by circles of correspondents in order to enable the reader to follow the dialogues of the partners. It has been decided that each exchange of letters – if edited on paper – will be published in its entirety.

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